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Application of Diagonalised Rhotrix to Solution of System of Differential Equations

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Abstract

Since the introduction of rhotrices, there are so many studies on the concept in relations to matrices, abstract structures, quadratic forms etc. The choice of diagonalization of rhotrix to solve the systems of differential equation was eminent. In this paper, the eigenvalues, eigenvectors and diagonalization of rhotrices were presented. A brief review of the system of ordinary differential equation and its solution was also presented; finally, various systems of ordinary differential equations were examined with regard to diagonalized rhotrices.

Keywords: *Rhotrices, Diagonalized Rhotrix, Eigenvalues of Rhotrix, Eigenvectors of Rhotrix, Differential Equations*

1. Introduction

The concept of rhotrix was first conceive and presented for mathematical enrichment, by (Ajibade, 2003). Rhotrix which is relatively a new paradigm of matrix theory, whose goal is central on representation of arrays of numbers in rhomboid mathematical form, was an extension of ideas on matrix tertions and matrix noitroits proposed by (Atnasson and Shannon, 1998).

(Sani, 2004) proposed an alternative method for multiplication of rhotrix, the method establishes some relationships between rhotrix and matrices. The row-column multiplication of high dimensional rhotrices were also presented by (Sani, 2007), where he presents the row-column multiplication of rhotrix that are of high dimension. This is an extension of some multiplication carried out on rhotrix of dimension three, considered to be the base rhotrix. (Kaurangini and Sani, 2007) presented the idea of rhotrix construction by using some ideas of a Hilbert matrix. The rhotrix was shown to have similar properties with a Hilbert matrix; furthermore it was shown that an isomorphism exists, which relates such rhotrices with Hilbert matrices. (Aminu, 2010) defined the n-dimensional main rhotrix column and row vectors respectively and presented them in short form as $\left\langle x^{n1}\right\rangle$ and $\left\langle x^{1n}\right\rangle$. Similarly. The non-main rhotrix column and row vectors were defined and represents them in short form as $\left\langle y^{n-11}\right\rangle$ and $\left\langle y^{1n-1}\right\rangle$.

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(Usaini and Mohammed, 2012) examined the rhotrix eigenvalue eigenvector problem (REP), as the extension of matrix eigenvalue eigenvector that can be seen in (Lipsehutz and Lipson, 2001) which was first introduced by (Aminu, 2010), where they presented some properties of rhotrix eigenvalues and eigenvectors considering the numerous applications of matrix eigenvector-eigenvalue problem in areas of Applied Mathematics as well as the diagonalization problem in terms of rhotrices (RDP). (Satyam et al., 2023) explores the mathematical framework of rhotrices to constructing balanced incomplete block designs (BIRD). BIRD play a pivotal role in experimental design, facilitating efficient and structured data collection.

The interest of this research aroused due to the similarities in properties between rhotrices and matrices. In this work the concept of diagonalization in matrix theory was adopted so as to presents an intuitive imagination of diagonalization of rhotrix.

2. Related Works

2.1 Systems of ordinary differential equations

Systems of ordinary differential equations have numerous application, this section presents systems of ordinary differential equations some of the basic concepts about systems of ordinary differential equation and their solutions as discuses by (Kneyszig, 2006).

The general system of the first order differential equation is given below:-

$$
\begin{aligned}
Y_1' &= f_1(t, y_1, \ldots, y_n) \\
Y_2' &= f_2(t, y_1, \ldots, y_n) \\
&\ldots \qquad\qquad\qquad\qquad\qquad\qquad (2.1) \\
Y_n' &= f_n(t, y_1, \ldots, y_n)
\end{aligned}
$$

Where $y_1, y_2, \ldots, y_n$ are unknown functions of t

We can write the system (2.1) as a vector equation by introducing the column vectors y = $(y_1,\ y_2, \ldots, y_n)^T$ and f = $(f_1,\ f_2,\ \ldots, f_n)^{\mathrm{T}}$ (where T means transposition and save us the space that would be needed for writing y and f as column). This gives

$$y' = f(t, y)$$

A solution of equations (2.1) on some interval a < t <b is a set of *n* differentiable function

$$y_1 = h_1(t), y_2 = h_2(t), \ldots, y_n = h_n(t)$$

On a < t <b that satisfy (1) throughout this interval. In vector form, by introducing the "solution vector" $h = (h_1, h_2,\ \ldots\ , h_n)^{\mathrm{T}}$ (a column vector) we can write

$$y = h(t)$$

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2.2 LINEAR SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS

Extending the notion of a linear ordinary differential equations then (2.1) is called a linear system, if it is linear in $y_1, \ldots, y_n$: that is, if it can be written

$$y_1' = a_{11}(t)y_1 + \ldots + a_{1n}(t)y_n + g_1(t)$$

$$y_2' = a_{21}(t)y_1 + \ldots + a_{2n}(t)y_n + g_2(t)$$

$$\ldots \qquad (2.2)$$

$$y_n' = a_{n1}(t)y_1 + \ldots + a_{nn}(t)y_n + g_n(t)$$

As a vector equation this becomes

$$y' = Ay + g \qquad (2.3)$$

This system is called homogenous if g=0, that is

$$y' = Ay. \qquad (2.4)$$

If $g \neq 0$, then (2.2) is called non homogeneous.

The general solution of the homogenous system (2.4) on some interval j means a linearly independent set of *n* solutions $y^{(1)}, \ldots, y^{(n)}$ of (2.4) on that interval; and (2.5) as a corresponding linear combination

$$y = c_1 y^{(1)} + \ldots + c_n y^{(n)} \qquad (2.5)$$

where $c_1, \ldots, c_n$ are arbitrary constants

3. METHODOLOGY

3.1 EIGENVALUES AND EIGENVECTORS OF A RHOTRIX

Let *A* be an n-dimensional rhotrix, then an eigenvalues of *A* is a scalar λ such that the vector equation

$$Ax = \lambda x \qquad (3.1)$$

has a trivial solution, that is, a solution $x \neq 0$, which is called an eigenvectors of *A*, corresponding to that eigenvalues λ. Where *x* (eigenvectors) are column vectors

(3.1) can be written as

$$(A - \lambda I)x = 0 \qquad (3.2)$$

where I is an n-dimensional identity rhotrix, x is a column rhotrix and λ is a scalar quantity. This homogeneous system has a non-trivial solution if and only if $\det(A - \lambda I) = 0$

Now, (3.1) in rhotrix form is given as

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$$\left\langle \begin{array}{c} a_{11} \\ a_{21} \quad c_{11} \quad a_{12} \\ - \quad - \quad - \quad - \quad - \\ a_{t1} \quad - \quad - \quad - \quad - \quad - \quad a_{1t} \\ - \quad - \quad - \quad - \quad - \\ a_{1t-1} \quad c_{t-1} \quad a_{t-1t} \\ a_{1-t} \end{array} \right\rangle \left\langle \begin{array}{c} x_1 \\ x_2 \quad 0 \quad 0 \\ - \quad - \quad - \quad - \quad - \\ x_t \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \\ - \quad - \quad - \quad - \quad - \\ 0 \quad 0 \quad 0 \\ 0 \end{array} \right\rangle = \lambda \left\langle \begin{array}{c} x_1 \\ x_2 \quad 0 \quad 0 \\ - \quad - \quad - \quad - \quad - \\ x_t \quad - \quad - \quad - \quad - \quad - \quad 0 \\ - \quad - \quad - \quad - \quad - \\ 0 \quad 0 \quad 0 \\ 0 \end{array} \right\rangle$$

EXAMPLE 3.1:- Find the eigenvalues and eigenvectors of the rhotrix A.

$$A = \left\langle \begin{array}{c} -2 \\ -2 \quad 0 \quad 6 \\ 5 \end{array} \right\rangle$$

Solution.

The rhotrix A is a three dimensional rhotrix, this implies that it will have 3 roots (eigenvalues).

$\det(A - I\lambda) = 0$ (i.e. the characteristic equation)

$$\det\left[\left\langle \begin{array}{c} -2 \\ -2 \quad 0 \quad 6 \\ 5 \end{array} \right\rangle - \lambda \left\langle \begin{array}{c} 1 \\ 0 \quad 1 \quad 0 \\ 1 \end{array} \right\rangle \right] = 0$$

$$\det\left[\left\langle \begin{array}{c} -2 \\ -2 \quad 0 \quad 6 \\ 5 \end{array} \right\rangle - \left\langle \begin{array}{c} \lambda \\ 0 \quad \lambda \quad 0 \\ \lambda \end{array} \right\rangle \right] = 0$$

$$\det \left\langle \begin{array}{c} -2-\lambda \\ -2 \quad -\lambda \quad 6 \\ 5-\lambda \end{array} \right\rangle = 0$$

Implies$(-\lambda)[(-2-\lambda)(5-\lambda)+12] = 0$

$(-\lambda)\left[-10 + 2\lambda - 5\lambda + \lambda^2 + 12\right] = 0$

$(-\lambda)\left[\lambda^2 - 3\lambda + 2\right] = 0$

$(-\lambda)(\lambda - 1)(\lambda - 2) = 0$

Which gives the real distinct eigenvalues $\lambda_1 = 0$, $\lambda_2 = 1$, $\lambda_3 = 2$

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To find the corresponding eigenvectors, it follow

For λ= 1

$$\left\langle \begin{array}{ccc} & -3 & \\ -2 & 0 & 6 \\ & 4 & \end{array} \right\rangle \left\langle \begin{array}{ccc} & x_1 & \\ x_2 & 0 & 0 \\ & 0 & \end{array} \right\rangle = 0$$

$$-3x_1 + 6x_2 = 0 \qquad (3.3)$$

$$-2x_1 + 4x_2 = 0 \qquad (3.4)$$

$x_1 = [2,1]^T$

For λ = 2

$$\left\langle \begin{array}{ccc} & 0 & \\ -2 & -1 & 6 \\ & 3 & \end{array} \right\rangle \left\langle \begin{array}{ccc} & x_1 & \\ x_2 & 0 & 0 \\ & 0 & \end{array} \right\rangle = 0$$

$$6x_2 = 0 \qquad (3.5)$$

$$-2x_1 + 3x_2 = 0 \qquad (3.6)$$

$\Rightarrow -2x_1 = 3x_2 \; and \; x_1 = 3/2x_2$

$x_2 = [3/2,1]^T$

EXAMPLE 3.2:- Find the eigenvalues and eigenvectors of the rhotrix A

$$A = \left\langle \begin{array}{ccccc} & & 4 & & \\ & 2 & 0 & 1 & \\ 1 & 0 & 5 & 0 & -1 \\ & 1 & 0 & 2 & \\ & & 2 & & \end{array} \right\rangle$$

Solution

The rhotrix A is a 5-dimensional rhotrix thus it has 5 roots (eigenvalues).

The characteristic equation is

$\det(A - \lambda I) = 0$

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$$\det\left\langle \begin{array}{ccccc} & & 4-\lambda & & \\ & 2 & -\lambda & 1 & \\ 1 & 0 & 5-\lambda & 0 & -1 \\ & 1 & -\lambda & 2 & \\ & & 2-\lambda & & \end{array} \right\rangle = 0$$

$$[(4-\lambda)\big((5-\lambda)(2-\lambda)+2\big)-(2(2-\lambda)+2)-(2-(5-\lambda))]\big(\lambda^2 - 0\big)=0$$

$$\Big[(4-\lambda)\Big(10-5\lambda-2\lambda+\lambda^2+2\Big)-(4-2\lambda+2)-(2-5+\lambda)\Big]\lambda^2 = 0$$

$$\Big[(4-\lambda)\Big(12-7\lambda+\lambda^2\Big)-(-2\lambda+6)-(\lambda-3)\Big]\lambda^2 = 0$$

$$(4\lambda^2-28\lambda+48-\lambda^3+7\lambda^2-12\lambda+2\lambda-6-\lambda+3)\lambda^2 = 0$$

$$(-\lambda^3+11\lambda^2-39\lambda+45)\lambda^2 = 0$$

either $\lambda^2 = 0$ or $-\lambda^3+11\lambda^2-39\lambda+45=0$

$\Rightarrow \lambda = 0$ (twice)

Then, $(\lambda-3)(\lambda-3)(\lambda-5)=0$

$\Rightarrow \lambda_1 = 3, \lambda_2 = 3, \lambda_3 = 5$

To find the corresponding eigenvectors, it follows

$for\ \lambda = 3$

$$\left\langle \begin{array}{ccccc} & & 1 & & \\ & 2 & -3 & 1 & \\ 1 & 0 & 2 & 0 & -1 \\ & 1 & -3 & -2 & \\ & & -1 & & \end{array} \right\rangle \left\langle \begin{array}{ccccc} & & x_1 & & \\ & x_2 & 0 & 0 & \\ x_3 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & \\ & & 0 & & \end{array} \right\rangle = 0$$

$x_1 + x_2 - x_3 = 0$ (3.7)

$2x_1 + 2x_2 - 2x_3 = 0$ (3.8)

From (3.7)

$x_1 = x_3 - x_2$

Let $x_3 = 0$ then

$x_1 = -x_2$

$let\ x_1 = 1, \quad \Rightarrow x_2 = -1$

$x_1 = (1, -1, 0)^T$

From (3.8)

Let $x_2 = 0, \quad \Rightarrow 2x_1 = 2x_3 \Rightarrow x_1 = x_3$

Let $x_1 = 1, \quad \Rightarrow x_3 = 1$

$x_2 = (1, 0, 1)^T$

$for\ \lambda = 5$

$$\left\langle \begin{array}{ccccc} & & 1 & & \\ & 2 & -5 & 1 & \\ -1 & 0 & 2 & 0 & -1 \\ & 1 & -5 & -2 & \\ & & -3 & & \end{array} \right\rangle \left\langle \begin{array}{ccccc} & & x_1 & & \\ & x_2 & 0 & 0 & \\ x_3 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & \\ & & 0 & & \end{array} \right\rangle = 0$$

$-x_1 + x_2 - x_3 = 0$ (3.9)

$2x_1 - 2x_3 = 0$ (3.10)

From (3.9)

$x_1 = x_3$

Put $x_1 = x_3$ then from (3.10) it comes out as

$-x_1 + x_2 - x_3 = 0 \quad \Rightarrow x_2 = 2x_3$

Let $x_3 = 1 \ \Rightarrow x_1 = 1 \ \ and\ x_2 = 2$

$x_3 = (1, 2, 1)^T$

4. RESULT AND DISCUSSIONS

4.1 DIAGONALIZATION OF RHOTRIX

If an n – dimensional rhotrix A has a basis of eigenvectors, then

$$D = P^{-1}AP \tag{4.1}$$

Is diagonal rhotrix whose diagonal entries are the eigenvalue of A corresponding to the eigenvectors appearing in P. Here P is the rhotrix with these eigenvectors as column vectors. Hence A is diagonalizable if

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$$A = PDP^{-1} \tag{4.2}$$

The above relation illustrates that once such λ_i and corresponding x_i $\{i = 1,2,\dots\}$ are determined, then the rhotrices P and D can easily be defined.

In fact, λ_i is called eigenvalues and V_i is called an eigenvector of the corresponding eigenvalue of A. It also gives an insight on how to construct the invertible P when A is diagonizable.

Example 4.1:- Consider the rhotrix below and diagonalize it.

$$A = \left\langle \begin{array}{ccc} & -2 & \\ -2 & 0 & 6 \\ & 5 & \end{array} \right\rangle$$

$$D = P^{-1}AP$$

But, from example (3.1), the eigenvalues of A and its corresponding eigenvectors were determined as: $\lambda_1 = 1$ and $\lambda_2 = 2$ (eigenvalues) and $x_1 = [2,1]^{\mathrm{T}}$ and $x_2 = [3/2,1]^{\mathrm{T}}$.

Now,

$$P = \left\langle \begin{array}{ccc} & 2 & \\ 1 & 0 & 3/2 \\ & 1 & \end{array} \right\rangle \text{ and } P^{-1} = \left\langle \begin{array}{ccc} & 2 & \\ -2 & 0 & -3 \\ & 4 & \end{array} \right\rangle$$

$$D = \left\langle \begin{array}{ccc} & 2 & \\ -2 & 0 & -3 \\ & 4 & \end{array} \right\rangle \left\langle \begin{array}{ccc} & -2 & \\ -2 & 0 & 6 \\ & 5 & \end{array} \right\rangle \left\langle \begin{array}{ccc} & 2 & \\ 1 & 0 & 3/2 \\ & 1 & \end{array} \right\rangle = \left\langle \begin{array}{ccc} & 1 & \\ 0 & 0 & 0 \\ & 2 & \end{array} \right\rangle$$

Hence, A is diagonalizable.

Example 4.2:- Diagonalize the following rhotrix.

$$A = \left\langle \begin{array}{ccccc} & & 4 & & \\ & 2 & 0 & 1 & \\ 1 & 0 & 5 & 0 & -1 \\ & 1 & 0 & 2 & \\ & & 2 & & \end{array} \right\rangle$$

Also from example (3.2), the eigenvalues and the corresponding eigenvectors of the above rhotrix are

$\lambda_1 = 3,\ \lambda_2 = 3$ and $\lambda_3 = 5$ (eigenvalues)

$x_1 = (1,-1,0)^{\mathrm{T}}$, $x_2 = (1,0,1)^{\mathrm{T}}$ and $x_3 = (1,2,1)^{\mathrm{T}}$then

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$$P=\left\langle\begin{array}{ccccc} & & 1 & & \\ & -1 & 0 & 1 & \\ 0 & 0 & 0 & 0 & 1 \\ & 1 & 0 & 2 & \\ & & 1 & & \end{array}\right\rangle \quad \text{and} \quad P^{-1}=\left\langle\begin{array}{ccccc} & & 1 & & \\ & -1/2 & 0 & 0 & \\ 1/2 & 0 & -1/2 & 0 & -1 \\ & 1/2 & 0 & 3/2 & \\ & & 1 & & \end{array}\right\rangle$$

Then, $D = P^{-1}AP$

$$D=\left\langle\begin{array}{ccccc} & & 1 & & \\ & -1 & 0 & 1 & \\ 0 & 0 & 0 & 0 & 1 \\ & 1 & 0 & 2 & \\ & & 1 & & \end{array}\right\rangle\left\langle\begin{array}{ccccc} & & 4 & & \\ & 2 & 0 & 1 & \\ 1 & 0 & 5 & 0 & -1 \\ & 1 & 0 & 2 & \\ & & 2 & & \end{array}\right\rangle\left\langle\begin{array}{ccccc} & & 1 & & \\ & -1/2 & 0 & 0 & \\ 1/2 & 0 & -1/2 & 0 & -1 \\ & 1/2 & 0 & 3/2 & \\ & & 1 & & \end{array}\right\rangle$$

$$D=\left\langle\begin{array}{ccccc} & & 3 & & \\ & 0 & 0 & 0 & \\ 0 & 0 & 3 & 0 & 0 \\ & 0 & 0 & 0 & \\ & & 5 & & \end{array}\right\rangle$$

4.2 APPLICATION OF DIAGONALIZATION TO SOLUTION OF SYSTEM OF DIFFERENTIAL EQUATION

This section presents the main work of this research, that is, solving the system of Differential Equation using diagonalization of rhotrices

Example 4.2.1:- Consider the following system of linear differential equation

$$x_1' = -2x_1 + 6x_2 \qquad (4.3)$$
$$x_2' = -2x_1 + 5x_2$$

Rewrite the original system unto vectors and rhotrices.

$x' = Ax$

where

$$A=\left\langle\begin{array}{ccc} & -2 & \\ -2 & 0 & 6 \\ & 5 & \end{array}\right\rangle, \quad x(t)=\left\langle\begin{array}{ccc} & x_1(t) & \\ x_2(t) & 0 & 0 \\ & 0 & \end{array}\right\rangle$$

The above rhotrix A has a diagonalization

$$\left\langle \begin{array}{ccc} & -2 & \\ -2 & 0 & 6 \\ & 5 & \end{array} \right\rangle = \left\langle \begin{array}{ccc} & 2 & \\ 1 & 0 & 3/2 \\ & 1 & \end{array} \right\rangle \left\langle \begin{array}{ccc} & 1 & \\ 0 & 0 & 0 \\ & 2 & \end{array} \right\rangle \left\langle \begin{array}{ccc} & 2 & \\ -2 & 0 & -3 \\ & 4 & \end{array} \right\rangle$$

The general solution is

$$x(t) = \left\langle \begin{array}{ccc} & 2 & \\ 1 & 0 & 3/2 \\ & 1 & \end{array} \right\rangle \left\langle \begin{array}{ccc} & C_1 e^{t} & \\ C_2 e^{2t} & 0 & 0 \\ & 0 & \end{array} \right\rangle$$

Or equivalently

$$x_1(t) = 2C_1 e^{t} + 3/2 C_2 e^{2t}$$

$$x_2(t) = C_1 e^{t} + C_2 e^{2t}$$

Example 4.2.2:- Use diagonalization to solve the following linear system of ordinary differential equations

$$x_1 = 4x_1 + x_2 - x_3$$
$$x_2 = 2x_1 + 5x_2 + 2x_3 \qquad (4.4)$$
$$x_3 = x_1 + x_2 + 2x_3$$

The coefficient rhotrix has a diagonalization such that

$$A = PDP^{-1}$$

$$\left\langle \begin{array}{ccccc} & & 4 & & \\ & 2 & 0 & 1 & \\ 1 & 0 & 5 & 0 & -1 \\ & 1 & 0 & 2 & \\ & & 2 & & \end{array} \right\rangle = \left\langle \begin{array}{ccccc} & & 1 & & \\ & -1 & 0 & 1 & \\ 0 & 0 & 0 & 0 & 1 \\ & 1 & 0 & 2 & \\ & & 1 & & \end{array} \right\rangle \left\langle \begin{array}{ccccc} & & 3 & & \\ & 0 & 0 & 0 & \\ 0 & 0 & 3 & 0 & 0 \\ & 0 & 0 & 0 & \\ & & 5 & & \end{array} \right\rangle \left\langle \begin{array}{ccccc} & & 1 & & \\ & -1/2 & 0 & & \\ 1 & 0 & -1/2 & 0 & -1 \\ & 1/2 & 0 & 3/2 & \\ & & -1/2 & & \end{array} \right\rangle$$

The general solution is

$$x(t) = \left\langle \begin{array}{ccccc} & & 1 & & \\ & -1 & 0 & 1 & \\ 0 & 0 & 0 & 0 & 1 \\ & 1 & 0 & 2 & \\ & & 1 & & \end{array} \right\rangle \left\langle \begin{array}{ccccc} & & C_1 e^{3t} & & \\ & C_2 e^{3t} & 0 & 0 & \\ C_3 e^{5t} & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & \\ & & 0 & & \end{array} \right\rangle$$

Or equivalently

$$x_1(t) = (C_1 + tC_2)e^{3t} + C_3 e^{5t}$$
$$x_2(t) = -C_1 e^{3t} + 2C_3 e^{5t}$$
$$x_3(t) = C_2 e^{3t} + C_3 e^{5t}$$

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5. CONCLUSION

In conclusion, an eigenvalues and eigenvectors of a rhotrix were clearly defined, and these are basics to diagonalization of rhotrix; furthermore this paper presents the concept of diagonalization of rhotrix. Moreover, the concept had been used to solve a system of ordinary differential equations

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